Experimental Determination of the Mass of a Neutron

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The neutron is fundamental to our understanding of matter and the universe at large. In the present study, we aimed to experimentally determine the mass of the neutron m_n using a hydrogen-deuterium lamp by taking advantage of the fact one can observe splitting in the spectral lines associated with the difference in nuclear mass of hydrogen and deuterium. We measured m_n to be 1.685×10^{-27} kg $\pm\, 6.59 \times 10^{-30}$ kg, which fell within 0.6% of the accepted value. Additionally, we were able to determine the identity of a "mystery lamp" to be a helium discharge lamp based on the wavelengths and intensities of spectral peaks observed in its emission spectrum. We have also suggested a few ways in which this experiment could be improved, such as more precise alignment of the optics to produce a higher signal-to-noise ratio. Overall, our results serve as strong confirmation of the accepted value for m_n and garner support for its underlying theory. Lastly, our value facilitates numerical calculations in various fields across the sciences and industry.

I. INTRODUCTION

Since its discovery in 1932 by English physicist James Chadwick, the neutron has played a fundamental role in our understanding of the atom. In particular, the discovery was the first clear evidence in support of the proton-neutron model of the nucleus, which was to soon replace the nuclear electrons hypothesis present at the time. In a series of experiments investigating the properties of radiation excited in beryllium, Chadwick set out with the goal of verifying the existence of a neutral particle - an uncharted particle with the same mass as the proton. His experiments were successful as he was able to show not only that the neutron exists but that it had a mass 0.1% more than the proton. As a consequence, Chadwick was awarded the Nobel Prize in 1935 for his work [1].

It is important to know the numerical value of the mass of the neutron, a fundamental constant of nature, because the quantitative predictions of the basic theories of physics depend on the numerical values of the constants that appear in those theories. Furthermore, determining the numerical values of physical constants can help to asses the overall correctness of the basic theories themselves.

Knowledge of the physical properties of neutrons has many practical applications. For example, since the neutron plays an important role in many nuclear reactions, having the numerical value for its mass has played a critical role in the development of nuclear reactors and nuclear weapons. In particular, the absorption of neutrons by radioactive elements such as uranium-235 and plutonium-239 induces nuclear fission, which is at the heart of nuclear-based power sources. Additionally, neutron radiation can also be exploited in various medical therapies, such as delivering energy to cancerous tissues at a high rate [2].

In this experiment, we set out to perform spectroscopy on a hydrogen-deuterium mixture using a discharge lamp, a calibrated spectrometer, and a photomultiplier. We aimed to observe the Balmer series, and in particular, see a splitting in the spectral lines associated with the difference in nuclear mass of hydrogen and deuterium. Using the Bohr model, we sought to extract a measurement of the neutron mass from the observation of this splitting.

II. THEORY

Between 1913, and 1915, Niels Bohr developed a quantitative atomic model for the hydrogen atom: atoms consisting of a nucleus with positive charge $+\mathcal{Z}e$, where $\mathcal{Z}$ is the atomic number and e is the elementary charge. By applying Newton's second law to the assumed circular orbit of the electron, Bohr was able to write an expression for the total energy. Now, the key feature of the Bohr model is the assumption that the orbital angular momentum of the electron is quantized such that its magnitude can only be integer multiples of Plank's constant divided by 2π. With these facts in mind, Bohr was able to say that only certain values of the energy are permitted. In symbols,

$$E_n = -\frac{1}{2}\frac{m\mathcal{Z}^2e^4}{(4\pi\varepsilon_0)^2\hbar^2}\frac{1}{n^2} = -(13.6 \text{ eV})\frac{\mathcal{Z}^2}{n^2} \tag{1}$$

where m is the mass of electron, $\mathcal{Z}$ is the number of protons in the nucleus ($\mathcal{Z} = 1$ for hydrogen) and e is the charge of the electron. Beyond the several constants, we see that since n, the quantum number, is an integer, the energy is also quantized and depends on the quantum number: the higher the n, the higher the energy. Thus, an electron with a quantum number > 1 can lower its energy by changing to an orbit with a lower value of n, an orbit closer to the nucleus. Such a jump is called a transition and, in this process, the electron lowers its energy, which appears as the energy of a photon emitted by the atom. In particular, a transition from a state of

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higher energy to one of lower energy should result in the emission of radiation with energy

$$E_i - E_f = \frac{hc}{\lambda} = \frac{m\mathcal{Z}^2 e^4}{(4\pi\varepsilon_0)^2\hbar^2}\left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right]. \tag{2}$$

The discrete emission spectrum of hydrogen therefore corresponds to electrons' transitions to lower energy levels. For historical reasons, the transition to the $n = 2$ state from $n > 2$ higher states is called the Balmer series and is responsible for the visible light spectrum of hydrogen. It is easy to show that the first few Balmer series wavelengths are

$$\begin{aligned} \lambda_1 &= 656.9 \text{ nm} \\ \lambda_2 &= 486.5 \text{ nm} \\ \lambda_3 &= 434.0 \text{ nm} \\ \lambda_4 &= 410.6 \text{ nm}. \end{aligned}$$

Initial experimental results lead Bohr to revised his postulates, now requiring that the total angular momentum, the combined angular momentum of both the electron and the nucleus, be quantized in units of $\hbar$. The result of this requirement is that the electron mass m must be replaced with the reduced mass μ_H:

$$\mu_H = \frac{m_e m_p}{m_e + m_p}$$

where m_e is the electron mass and m_p is the mass of the proton.

The inclusion of the nuclear mass into the equation for special energies suggests that different isotopes of the same element should show different spectra because these atoms would have different reduced mass. For example, the reduced mass should the deuterium system is given by

$$\mu_D = \frac{m_e m_d}{m_e + m_D}$$

where m_D is now the mass of deuterium. Thus, we can take advantage of the fact that the emission spectra differ. First, from eq. (2) we see that the wavelength of light emitted from a particular transition is inversely proportional to the reduced mass:

$$1/\lambda \propto \mu_X \tag{3}$$

Because of this proportionality, the nuclear mass of hydrogen and deuterium, m_e and m_D are related to different wavelengths of light produced by the two isotopes:

$$\frac{\lambda_H - \lambda_D}{\lambda_H} = \frac{\Delta\lambda}{\lambda_H} = \frac{1/\mu_H - 1/\mu_D}{1/\mu_H} = \frac{1/m_p - 1/m_d}{1/\mu_H} \tag{4}$$

Since $\mu_H \approx m_e$ to an excellent approximation, after some simplifications, eq. (4) becomes

$$\frac{\Delta\lambda}{\lambda_H} = \frac{m_e}{m_p} - \frac{m_e}{m_D} \tag{5}$$

Lastly, since we know that the only difference between the nucleus of hydrogen and the nucleus of deuterium is an additional neutron (*i.e.*, $m_n = m_D - m_p$) we can produce a final expression for the mass of the neutron:

$$m_n = \frac{m_e}{\left(\frac{m_e}{m_p} - \frac{\Delta\lambda}{\lambda_H}\right)} - m_p. \tag{6}$$

III. APPARATUS

To begin our experiment, we assembled the apparatus schematized in figure 2. To briefly summarize the apparatus, a hydrogen-deuterium gas discharge lamp was positioned in front of a convex lens to focus the light through a slit of adjustable width on the photomultiplier (PMT) complex. Inside the PMT assembly was a series of mirrors and a rotating diffraction grating, whose ultimate effect was to finely adjust the wavelength of light reaching the PMT. Changes in the intensity of light reaching the PMT corresponded to changes in voltage registered by the PMT, which was continuously sampled and recorded with data acquisition software developed in-house.

For each Balmer series line, we first scanned ± 2 nm around the suspected wavelength of the line to locate and identify peaks. Once a peak was located, we sampled it 12 times (5 trials for the 4th line), marking the wavelengths that corresponded to two particular sampling times. To speed up data collection, we scanned over the Balmer lines in both directions, *i.e.*, moving in the direction of increasing and decreasing wavelength.

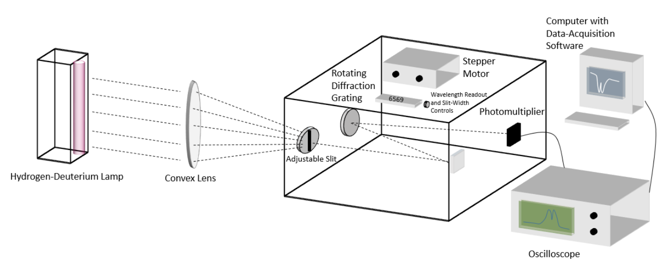

FIG. 1: A schematic of the apparatus used to observe the splitting in the spectral lines associated with the difference in nuclear mass of hydrogen and deuterium .

To determine the identity of a "mystery lamp", we replaced the HD lamp with the mystery sample and sampled its intensity as the wavelength ranged from approximately 480 nm to 590 nm.

IV. OBSERVATIONS AND ANALYSIS

The primary goal of this experiment was to measure the mass of the neutron. To make such a measurement, we chose to determine the difference in wavelength between the two peaks associated with the Balmer lines of an HD lamp. During the lab period, we made measurements of the wavelength while recording marks to produce the conversion factor given in (7). Since the motor moved in a discrete manner, we suspected that we could measure the wavelength with an uncertainty of 0.05 Å. However, we could not be sure that the wavelength meter was properly calibrated correctly. Fortunately, we were only interested in a difference in wavelength, so such calibration were not considered further.

Since the motor was moving at a constant rate, we knew that the wavelengths were also changing at a constant rate. Furthermore, since we marked the wavelength at two different times, we could convert the plot of voltage vs. time to the voltage as a function of wavelength. In particular, to make such a conversion, for each trail we computed a conversion factor in the following way:

$$\frac{\lambda_2 - \lambda_1}{T_2 - T_1} = \frac{\Delta\lambda}{\Delta T} \tag{7}$$

where λ_i is the wavelength sampled at the position T_i in the array of data.

Using (6) to calculate the mass of the neutron, one can see that all but one term are published values with uncertainties small enough to be considered negligible for our purposes. Therefore, we were only tasked with calculating a $\Delta\lambda$. We measured this parameter from the raw data using MATLAB's curve fitting toolbox. In particular, we fit to the data a double Gaussian of the form

$$f(x) = a_1 e^{-\left(\frac{x-\mu_1}{c_1}\right)^2} + d_1 + a_2 e^{-\left(\frac{x-b_2}{c_2}\right)^2} + d_2. \tag{8}$$

In this case, $\Delta\lambda = |b_1 - b_2|$ since the parameter b_i corresponds to the mean or horizontal position (wavelength) of each Gaussian peak. A representative example of the fitted raw data is shown in figure 2. MATLAB's curve fitting toolbox also provided us with estimates of uncertainty in these quantities, and since uncertainties add in quadrature, we have that

$$\sigma_{\Delta\lambda}^2 = \sigma_{b_1}^2 + \sigma_{b_2}^2. \tag{9}$$

From (9), we could calculate the total uncertainty of the mass of the neutron for each trial using

$$\sigma_{m_n}^2 = \left(\frac{m_e}{\lambda_H \left(\frac{m_e}{m_p} - \frac{\Delta\lambda}{\lambda_H}\right)^2}\right)^2 \sigma_{\Delta\lambda}^2, \tag{10}$$

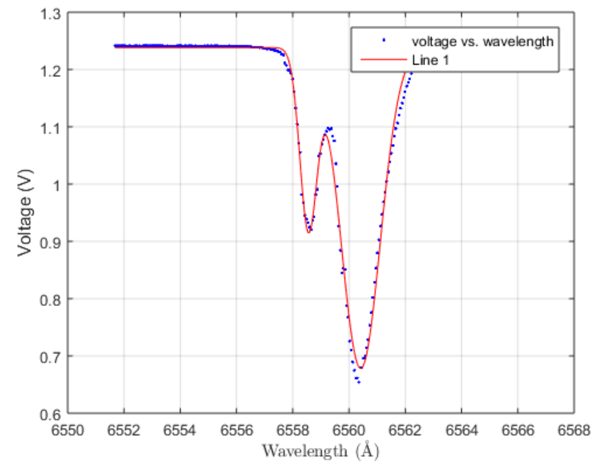

FIG. 2: A representative plot of voltage vs. wavelength fitted with a double Gaussian for the Balmer series wavelength line centered at 656 nm.

TABLE I: Best estimates for $\Delta\lambda$ for the four Balmer lines investigated.

Line #	$\Delta\lambda$ (Å)	Expected value (Å)
1	1.79 $\pm$ 0.0035	1.79
2	1.20 $\pm$ 0.0033	1.32
3	0.958 $\pm$ 0.013	1.18
4	6.31 $\pm$ 0.204	1.12

and average all trials for a given line using $1/\sigma^2$ weighting. Our analysis revealed that the best estimates of $\Delta\lambda$ for each Balmer lines are shown in table 1.

By (6), these measured differences in wavelengths correspond to the values for the mass of the neutron shown in table 2.

TABLE II: Best estimates for the mass of the neutron for the four Balmer lines investigated.

Line #	Neutron Mass (Kg)
1	1.685 $\times 10^{-27} \pm 6.59 \times 10^{-30}$
2	1.387 $\times 10^{-27} \pm 6.91 \times 10^{-30}$
3	6.78 $\times 10^{-30} \pm 9.12 \times 10^{-30}$
4	-2.59 $\times 10^{-27} \pm 4.61 \times 10^{-29}$

Initial examination of these results reveals that the first Balmer line produces the most reasonable result. Therefore, we take our best value of the mass of the neutron to be the value obtained from the first line

$$1.685 \times 10^{-27}\ \text{kg} \pm 6.59 \times 10^{-30}\ \text{kg}$$

Given the accepted value in the literature of 1.675×10^{-27} kg , we obtained approximately 0.6% error.

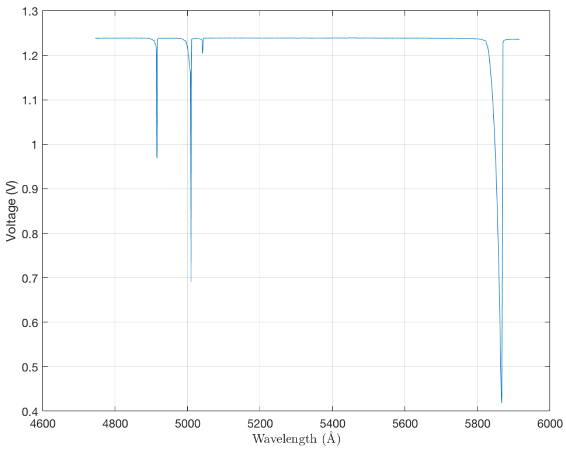

FIG. 3: The spectral scan from ≈ 4800 Å to 5900 Å of a mystery lamp determined to be a helium discharge lamp. Peaks are observed at approximately 492, 501, 504 and 587 nm.

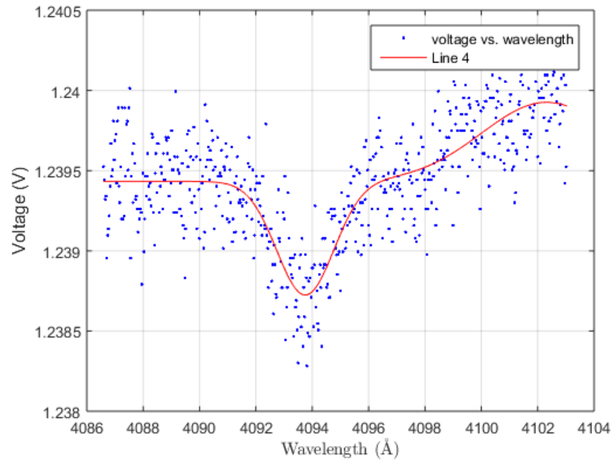

FIG. 4: A low signal-to-noise ratio in the fourth Balmer line made distinguishing peaks challenging. As a consequence, the results from the fourth Balmer line are to be discarded.

A secondary objective of this experiment was to determine the identity of a "mystery'" lamp, based on its emission spectra (figure 3). Table 3 summarizes the expected and observed peaks of the suspected helium discharge lamp. We observed all the expected spectral lines in our scan range. Furthermore, the predicted intensity of each peak matched the observed intensity in our plot. Therefore, we concluded that the mystery lamp was a helium discharge lamp.

TABLE III: Expected spectral peaks of a helium discharge lamps and the intensity of peaks observed at those wavelengths from the mystery lamp.

Wavelength (nm)	Observed Intensity
438.793	Out of our range
443.755	
447.148	
471.314	
492.193	medium
501.567	strong
504.774	weak
5887.562	strong
667.815	Out of Range

One major source of error and a challenging aspect of this experiment was the low signal-to-noise ration obtained with the third and fourth Balmer lines (figure 4). Our apparatus was such that increasing the slit-width would increase the signal intensity; however, there was a trade-off between signal intensity and signal resolution. Since the intensity of the signal produced by the third and fourth Balmer lines were comparatively minuscule, we were forced to increase the slit-width as to be able to observe a signal. As a consequence, it was really challenging, if not impossible, to distinguish between the two peaks in these cases. In fact, the results from the fourth Balmer line should be completely discarded because the separation between the peaks determined by MATLAB was almost 6 times too large, which produced a negative value for the mass of the neutron - this does not make sense, physically.

One way to improve the results for the third and fourth Balmer lines would be to spend more effort in systematically aligning the lamp and lens such that the intensity of the light entering the slit and reaching the PMT is increased. In this way, we could reduce the slit-width, giving us finer resolution to properly resolve the peak separation.

V. CONCLUSIONS AND OUTLOOK

In this experiment, we set out to measure the mass of the neutron using a hydrogen-deuterium lamp. We took advantage of the fact that the ratio of the mass of the proton to electron differs slightly from the ratio of the mass of deuterium atom and electron. The end consequence of of this fact was the emission of to two peaks separated by a calculated difference of wavelength at each Balmer line. By measuring this difference in wavelength, we could determine the mass of the neutron using the expression (6) given earlier. We had remarkable success in measuring the mass of the neutron using the first Balmer line, attaining only 0.6% error. As we moved through the Balmer series became more challenging to resolve, and therefore, our results were not as accurate for these conditions. Furthermore, we have identified potential sources of error that could be addressed if we were to pursue additional experiments. Overall, the results of this experiment are useful as they serve as a strong confirmation of the accepted value for m_n, and garner support for the underlying theory of the atom. Furthermore, our value will facilitate numerical calculations involved in the development of new technologies across various industries.

Acknowledgments

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[1] C. Purcell, *Electricity and Magnetism*, Cambridge University Press, 3rd edition, 2013.
[2] R. Reese, *University Physics*, Brooks Cole, 1st edition, 1999